Real-time Demand Forecasting for an Urban Delivery Platform

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Abstract

Meal delivery platforms like Uber Eats shape the landscape in cities around the world. This paper addresses forecasting demand into the short-term future. We propose an approach incorporating both classical forecasting and machine learning methods. Model evaluation and selection is adapted to demand typical for such a platform (i.e., intermittent with a double-seasonal pattern). The results of an empirical study with a European meal delivery service show that machine learning models become competitive once the average daily demand passes a threshold. As a main contribution, the paper explains how a forecasting system must be set up to enable predictive routing.

Keywords: demand forecasting, intermittent demand, machine learning, urban delivery platform

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1. Introduction

In recent years, many meal delivery platform providers (e.g., Uber Eats, GrubHub, DoorDash, Deliveroo) with different kinds of business models have entered the markets in cities around the world. A study by [26] estimates the global market size to surpass 20 billion Dollars by 2025. A common feature of these platforms is that they do not operate kitchens but focus on marketing their partner restaurants' meals, unifying all order related processes in simple smartphone apps, and managing the delivery via a fleet of either employees or crowd-sourced sub-contractors.

Various kind of urban delivery platforms (UDP) have received attention in recent scholarly publications. [29] look into heuristics to simultaneously optimize courier scheduling and routing in general, while [40] do so for the popular dial-a-ride problem and [56] investigate the effect of different fulfillment strategies in the context of urban meal delivery. [21] and [1] focus their research on the routing aspect, which is commonly modeled as a so-called vehicle routing problem (VRP).

Not covered in the recent literature is research focusing on the demand forecasting problem a UDP faces. Due to the customers' fragmented locations and the majority of the orders occurring ad-hoc for immediate delivery in the case of a meal delivery platform, forecasting demand for the near future (i.e., several hours) and distinct locations of the city in real-time is an essential factor in achieving timely fulfillment. In general, demand forecasting is a well-researched discipline with a decades-long history in scholarly journals as summarized, for example, by [19]. Even some meal delivery platforms themselves publish their practices: For example, [5] provide a general overview of supply and demand forecasting at Uber and benchmarks of the methods used while [38] investigate how extreme events can be incorporated.

The conditions such platforms face are not limited to meal delivery: Any entity that performs ad-hoc requested point-to-point transportation at scale in an urban area benefits from a robust forecasting system. Examples include ride-hailing, such as the original Uber offering, or bicycle courier services. The common characteristics are:

- **Geospatial Slicing**: Forecasts for distinct parts of a city in parallel

- **Temporal Slicing**: Forecasts on a sub-daily basis (e.g., 60-minute windows)
- **Order Sparsity**: The historical order time series exhibit an intermittent pattern
- **Double Seasonality**: Demand varies with the day of the week and the time of day

Whereas the first two points can be assumed to vary with the concrete application's requirements, it is the last two that pose challenges for forecasting a platform's demand: Intermittent demand (i.e., many observations in the historic order time series exhibit no demand at all) renders most of the commonly applied error metrics useless. Moreover, many of the established forecasting methods can only handle a single and often low seasonality (i.e., repeated regular pattern), if at all.

In this paper, we develop a rigorous methodology as to how to build and evaluate a robust forecasting system for an urban delivery platform (UDP) that offers ad-hoc point-to-point transportation of any kind. We implement such a system with a broad set of commonly used forecasting methods. We not only apply established (i.e., "classical") time series methods but also machine learning (ML) models that have gained traction in recent years due to advancements in computing power and availability of larger amounts of data. In that regard, the classical methods serve as benchmarks for the ML methods. Our system is trained on and evaluated with a dataset obtained from an undisclosed industry partner that, during the timeframe of our study, was active in several European countries and, in particular, in France. Its primary business strategy is the delivery of meals from upper-class restaurants to customers in their home or work places via bicycles. In this empirical study, we identify the best-performing methods. Thus, we answer the following research questions:

Q1: Which forecasting methods work best under what circumstances?
Q2: How do classical forecasting methods compare with ML models?
Q3: How does the forecast accuracy change with more historic data available?
Q4: Can real-time information on demand be exploited?
Q5: Can external data (e.g., weather data) improve the forecast accuracy?

To the best of our knowledge, no such study has yet been published in a scholarly journal.

The subsequent Section 2 reviews the literature on the forecasting methods included in the system. Section 3 introduces our forecasting system, and Section 4 discusses the results

obtained in the empirical study. Lastly, Section 5 summarizes our findings and concludes with an outlook on further research opportunities.

2. Literature Review

In this section, we review the specific forecasting methods that make up our forecasting system. We group them into classical statistics and ML models. The two groups differ mainly in how they represent the input data and how accuracy is evaluated.

A time series is a finite and ordered sequence of equally spaced observations. Thus, time is regarded as discrete and a time step as a short period. Formally, a time series Y is defined as $Y = \{y_t : t \in I\}$, or y_t for short, where I is an index set of positive integers. Besides its length $T = |Y|$, another property is the a priori fixed and non-negative periodicity k of a seasonal pattern in demand: k is the number of time steps after which a pattern repeats itself (e.g., $k = 12$ for monthly sales data).

2.1. Demand Forecasting with Classical Forecasting Methods

Forecasting became a formal discipline starting in the 1950s and has its origins in the broader field of statistics. [30] provide a thorough overview of the concepts and methods established, and [44] indicate business-related applications such as demand forecasting. These "classical" forecasting methods share the characteristic that they are trained over the entire Y first. Then, for prediction, the forecaster specifies the number of time steps for which he wants to generate forecasts. That is different for ML models.

2.1.1. Naïve Methods, Moving Averages, and Exponential Smoothing.

Simple forecasting methods are often employed as a benchmark for more sophisticated ones. The so-called naïve and seasonal naïve methods forecast the next time step in a time series, y_{T+1}, with the last observation, y_T, and, if a seasonal pattern is present, with the observation k steps before, y_{T+1-k}. As variants, both methods can be generalized to include drift terms in the presence of a trend or changing seasonal amplitude.

If a time series exhibits no trend, a simple moving average (SMA) is a generalization of the naïve method that is more robust to outliers. It is defined as follows: $\hat{y}_{T+1} = \frac{1}{h} \sum_{i=T-h}^{T} y_i$

where h is the horizon over which the average is calculated. If a time series exhibits a seasonal pattern, setting h to a multiple of the periodicity k suffices that the forecast is unbiased.

Starting in the 1950s, another popular family of forecasting methods, so-called exponential smoothing methods, was introduced by [13], [28], and [58]. The idea is that forecasts $\hat{y}_{T+1}$ are a weighted average of past observations where the weights decay over time; in the case of the simple exponential smoothing (SES) method we obtain: $\hat{y}_{T+1} = \alpha y_T + \alpha(1-\alpha)y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + \cdots + \alpha(1-\alpha)^{T-1} y_1$ where α (with $0 \leq \alpha \leq 1$) is a smoothing parameter.

Exponential smoothing methods are often expressed in an alternative component form that consists of a forecast equation and one or more smoothing equations for unobservable components. Below, we present a generalization of SES, the so-called Holt-Winters' seasonal method, in an additive formulation. ℓ_t, b_t, and s_t represent the unobservable level, trend, and seasonal components inherent in y_t, and β and γ complement α as smoothing parameters:

$$
\begin{aligned}
\hat{y}_{t+1} &= \ell_t + b_t + s_{t+1-k} \\
\ell_t &= \alpha(y_t - s_{t-k}) + (1-\alpha)(\ell_{t-1} + b_{t-1}) \\
b_t &= \beta(\ell_t - \ell_{t-1}) + (1-\beta) b_{t-1} \\
s_t &= \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma) s_{t-k}
\end{aligned}
$$

With b_t, s_t, β, and γ removed, this formulation reduces to SES. Distinct variations exist: Besides the three components, [22] add dampening for the trend, [45] provides multiplicative formulations, and [52] adds dampening to the latter. The accuracy measure commonly employed is the sum of squared errors between the observations and their forecasts.

Originally introduced by [2], [31] show how the Theta method can be regarded as an equivalent to SES with a drift term. We mention this method here only because [5] emphasize that it performs well at Uber. However, in our empirical study, we find that this is not true in general.

[35] introduce statistical processes, so-called innovations state-space models, to generalize the methods in this sub-section. They call this family of models ETS as they capture error,

trend, and seasonal terms. Linear and additive ETS models have a structure like so:

$$y_t = \vec{w} \cdot \vec{x}_{t-1} + \epsilon_t$$
$$\vec{x_t} = \boldsymbol{F}\vec{x}_{t-1} + \vec{g}\epsilon_t$$

y_t denote the observations as before while $\vec{x}_t$ is a state vector of unobserved components. ϵ_t is a white noise series and the matrix $\boldsymbol{F}$ and the vectors $\vec{g}$ and $\vec{w}$ contain a model's coefficients. Just as the models in the next sub-section, ETS models are commonly fitted with maximum likelihood and evaluated using information theoretical criteria against historical data. We refer to [34] for a thorough summary.

2.1.2. Autoregressive Integrated Moving Averages.

[7], [8], and more papers by the same authors in the 1960s introduce a type of model where observations correlate with their neighbors and refer to them as autoregressive integrated moving average (ARIMA) models for stationary time series. For a thorough overview, we refer to [9] and [12].

A time series y_t is stationary if its moments are independent of the point in time where it is observed. A typical example is a white noise ϵ_t series. Therefore, a trend or seasonality implies non-stationarity. [37] provide a test to check the null hypothesis of stationary data. To obtain a stationary time series, one chooses from several techniques: First, to stabilize a changing variance (i.e., heteroscedasticity), one applies a Box-Cox transformation (e.g., log) as first suggested by [6]. Second, to factor out a trend (or seasonal) pattern, one computes differences of consecutive (or of lag k) observations or even differences thereof. Third, it is also common to pre-process y_t with one of the decomposition methods mentioned in Sub-section 2.1.3 below with an ARIMA model then trained on an adjusted y_t.

In the autoregressive part, observations are modeled as linear combinations of its predecessors. Formally, an $AR(p)$ model is defined with a drift term c, coefficients ϕ_i to be estimated (where i is an index with $0 < i \leq p$), and white noise ϵ_t like so: $AR(p)$: $y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t$. The moving average part considers observations to be regressing towards a linear combination of past forecasting errors. Formally, a $MA(q)$ model is defined with a drift term c, coefficients θ_j to be estimated, and white noise terms ϵ_t (where j is an index with $0 < j \leq q$) as follows: $MA(q)$: $y_t =$

$c+\epsilon_t+\theta_1\epsilon_{t-1}+\theta_2\epsilon_{t-2}+\cdots+\theta_q\epsilon_{t-q}$. Finally, an $ARIMA(p,d,q)$ model unifies both parts and adds differencing where d is the degree of differences and the $'$ indicates differenced values: $ARIMA(p,d,q): \quad y'_t = c+\phi_1 y'_{t-1}+\cdots+\phi_p y'_{t-p}+\theta_1\epsilon_{t-1}+\cdots+\theta_q\epsilon_{t-q}+\epsilon_t$.

$ARIMA(p,d,q)$ models are commonly fitted with maximum likelihood estimation. To find an optimal combination of the parameters p, d, and q, the literature suggests calculating an information theoretical criterion (e.g., Akaike's Information Criterion) that evaluates the fit on historical data. [32] provide a step-wise heuristic to choose p, d, and q, that also decides if a Box-Cox transformation is to be applied, and if so, which one. To obtain a one-step-ahead forecast, the above equation is reordered such that t is substituted with $T+1$. For forecasts further into the future, the actual observations are subsequently replaced by their forecasts. Seasonal ARIMA variants exist; however, the high frequency k in the kind of demand a UDP faces typically renders them impractical as too many coefficients must be estimated.

2.1.3. Seasonal and Trend Decomposition using Loess.

A time series y_t may exhibit different types of patterns; to fully capture each of them, the series must be decomposed. Then, each component is forecast with a distinct model. Most commonly, the components are the trend t_t, seasonality s_t, and remainder r_t. They are themselves time series, where only s_t exhibits a periodicity k. A decomposition may be additive (i.e., $y_t = s_t + t_t + r_t$) or multiplicative (i.e., $y_t = s_t * t_t * r_t$); the former assumes that the effect of the seasonal component is independent of the overall level of y_t and vice versa. The seasonal component is centered around 0 in both cases such that its removal does not affect the level of y_t. Often, it is sufficient to only seasonally adjust the time series, and model the trend and remainder together, for example, as $a_t = y_t - s_t$ in the additive case.

Early approaches employed moving averages (cf., Sub-section 2.1.1) to calculate a trend component, and, after removing that from y_t, averaged all observations of the same seasonal lag to obtain the seasonal component. The downsides of this are the subjectivity in choosing the window lengths for the moving average and the seasonal averaging, the incapability of the seasonal component to vary its amplitude over time, and the missing handling of outliers.

The X11 method developed at the U.S. Census Bureau and described in detail by [18] overcomes these disadvantages. However, due to its background in economics, it is designed

primarily for quarterly or monthly data, and the change in amplitude over time cannot be controlled. Variants of this method are the SEATS decomposition by the Bank of Spain and the newer X13-SEATS-ARIMA method by the U.S. Census Bureau. Their main advantages stem from the fact that the models calibrate themselves according to statistical criteria without manual work for a statistician and that the fitting process is robust to outliers.

[16] introduce a seasonal and trend decomposition using a repeated locally weighted regression - the so-called Loess procedure - to smoothen the trend and seasonal components, which can be viewed as a generalization of the methods above and is denoted by the acronym STL. In contrast to the X11, X13, and SEATS methods, the STL supports seasonalities of any lag k that must, however, be determined with additional statistical tests or set with out-of-band knowledge by the forecaster (e.g., hourly demand data implies $k = 24 * 7 = 168$ assuming customer behavior differs on each day of the week). Moreover, the seasonal component's rate of change, represented by the ns parameter and explained in detail with Figure 4 in Section 3.5, must be set by the forecaster as well, while the trend's smoothness may be controlled via setting a non-default window size. Outliers are handled by assignment to the remainder such that they do not affect the trend and seasonal components. In particular, the manual input needed to calibrate the STL explains why only the X11, X13, and SEATS methods are widely used by practitioners. However, the widespread adoption of concepts like cross-validation (cf., Sub-section 2.2.2) in recent years enables the usage of an automated grid search to optimize the parameters. The STL's usage within a grid search is facilitated even further by its being computationally cheaper than the other methods discussed.

2.2. Demand Forecasting with Machine Learning Methods

ML methods have been employed in all kinds of prediction tasks in recent years. In this section, we restrict ourselves to the models that performed well in our study: Random Forest (RF) and Support Vector Regression (SVR). RFs are in general well-suited for datasets without a priori knowledge about the patterns, while SVR is known to perform well on time series data, as shown by [23] in general and [3] specifically for intermittent demand. Gradient Boosting, another popular ML method, was consistently outperformed by RFs, and artificial neural networks require an amount of data exceeding what our industry partner has by far.

2.2.1. Supervised Learning.

A conceptual difference between classical and ML methods is the format for the model inputs. In ML models, a time series Y is interpreted as labeled data. Labels are collected into a vector $\vec{y}$ while the corresponding predictors are aligned in an $(T-n) \times n$ matrix $\boldsymbol{X}$:

$$\vec{y} = \begin{pmatrix} y_T \\ y_{T-1} \\ \cdots \\ y_{n+1} \end{pmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} y_{T-1} & y_{T-2} & \cdots & y_{T-n} \\ y_{T-2} & y_{T-3} & \cdots & y_{T-(n+1)} \\ \cdots & \cdots & \cdots & \cdots \\ y_n & y_{n-1} & \cdots & y_1 \end{bmatrix}$$

The $m = T - n$ rows are referred to as samples and the n columns as features. Each row in $\boldsymbol{X}$ is "labeled" by the corresponding entry in $\vec{y}$, and ML models are trained to fit the rows to their labels. Conceptually, we model a functional relationship f between $\boldsymbol{X}$ and $\vec{y}$ such that the difference between the predicted $\vec{\hat{y}} = f(\boldsymbol{X})$ and the true $\vec{y}$ are minimized according to some error measure $L(\vec{\hat{y}}, \vec{y})$, where L summarizes the goodness of the fit into a scalar value (e.g., the well-known mean squared error [MSE]; cf., Section 3.4). $\boldsymbol{X}$ and $\vec{y}$ show the ordinal character of time series data: Not only overlap the entries of $\boldsymbol{X}$ and $\vec{y}$, but the rows of $\boldsymbol{X}$ are shifted versions of each other. That does not hold for ML applications in general (e.g., the classical example of predicting spam vs. no spam emails, where the features model properties of individual emails), and most of the common error measures presented in introductory texts on ML, are only applicable in cases without such a structure in $\boldsymbol{X}$ and $\vec{y}$. n, the number of past time steps required to predict a y_t, is an exogenous model parameter. For prediction, the forecaster supplies the trained ML model an input vector in the same format as a row $\vec{x}_i$ in $\boldsymbol{X}$. For example, to predict y_{T+1}, the model takes the vector $(y_T, y_{T-1}, ..., y_{T-n+1})$ as input. That is in contrast to the classical methods, where we only supply the number of time steps to be predicted as a scalar integer.

2.2.2. Cross-Validation.

Because ML models are trained by minimizing a loss function L, the resulting value of L underestimates the true error we see when predicting into the actual future by design. To counter that, one popular and model-agnostic approach is cross-validation (CV), as summarized, for example, by [24]. CV is a resampling technique, which ranomdly splits the samples

into a training and a test set. Trained on the former, an ML model makes forecasts on the latter. Then, the value of L calculated only on the test set gives a realistic and unbiased estimate of the true forecasting error, and may be used for one of two distinct aspects: First, it assesses the quality of a fit and provides an idea as to how the model would perform in production when predicting into the actual future. Second, the errors of models of either different methods or the same method with different parameters may be compared with each other to select the best model. In order to first select the best model and then assess its quality, one must apply two chained CVs: The samples are divided into training, validation, and test sets, and all models are trained on the training set and compared on the validation set. Then, the winner is retrained on the union of the training and validation sets and assessed on the test set.

Regarding the splitting, there are various approaches, and we choose the so-called k-fold CV, where the samples are randomly divided into k folds of the same size. Each fold is used as a test set once and the remaining $k-1$ folds become the corresponding training set. The resulting k error measures are averaged. A k-fold CV with $k = 5$ or $k = 10$ is a compromise between the two extreme cases of having only one split and the so-called leave-one-out CV where $k = m$: Computation is still relatively fast and each sample is part of several training sets maximizing the learning from the data. We adapt the k-fold CV to the ordinal stucture in $\boldsymbol{X}$ and $\vec{y}$ in Sub-section 3.3.

2.2.3. Random Forest Regression.

[11] introduce the classification and regression tree (CART) model that is built around the idea that a single binary decision tree maps learned combinations of intervals of the feature columns to a label. Thus, each sample in the training set is associated with one leaf node that is reached by following the tree from its root and branching along the arcs according to some learned splitting rule per intermediate node that compares the sample's realization for the feature specified by the rule to the learned decision rule. While such models are computationally fast and offer a high degree of interpretability, they tend to overfit strongly to the training set as the splitting rules are not limited to any functional form (e.g., linear) in the relationship between the features and the labels. In the regression case, it is common

to maximize the variance reduction I_V from a parent node N to its two children, $C1$ and $C2$, as the splitting rule. [11] formulate this as follows:

$$I_V(N) = \frac{1}{|S_N|^2} \sum_{i \in S_N} \sum_{j \in S_N} \frac{1}{2}(y_i - y_j)^2 - \left(\frac{1}{|S_{C1}|^2} \sum_{i \in S_{C1}} \sum_{j \in S_{C1}} \frac{1}{2}(y_i - y_j)^2 + \frac{1}{|S_{C2}|^2} \sum_{i \in S_{C2}} \sum_{j \in S_{C2}} \frac{1}{2}(y_i - y_j)^2 \right)$$

S_N, S_{C1}, and S_{C2} are the index sets of the samples in N, $C1$, and $C2$.

[27] and then [10] generalize this method by combining many CART models into one forest of trees where every single tree is a randomized variant of the others. Randomization is achieved at two steps in the training process: First, each tree receives a distinct training set resampled with replacement from the original training set, an idea also called bootstrap aggregation. Second, at each node a random subset of the features is used to grow the tree. Trees can be fitted in parallel speeding up the training significantly. For prediction at the tree level, the average of all the samples at a particular leaf node is used. Then, the individual values are combined into one value by averaging again across the trees. Due to the randomization, the trees are decorrelated offsetting the overfitting. Another measure to counter overfitting is pruning the tree, either by specifying the maximum depth of a tree or the minimum number of samples at leaf nodes.

The forecaster must tune the structure of the forest. Parameters include the number of trees in the forest, the size of the random subset of features, and the pruning criteria. The parameters are optimized via grid search: We train many models with parameters chosen from a pre-defined list of values and select the best one by CV. RFs are a convenient ML method for any dataset as decision trees do not make any assumptions about the relationship between features and labels. [25] use RFs to predict the hourly demand for water in an urban context, a similar application as the one in this paper, and find that RFs work well with time series type of data.

2.2.4. Support Vector Regression.

[55] and [54] introduce the so-called support vector machine (SVM) model, and [53] summarizes the research conducted since then. In its basic version, SVMs are linear classifiers, modeling a binary decision, that fit a hyperplane into the feature space of $\boldsymbol{X}$ to maximize the margin around the hyperplane seperating the two groups of labels. SVMs were popularized in the 1990s in the context of optical character recognition, as shown in [47].

[20] and [50] adapt SVMs to the regression case, and [49] provide a comprehensive introduction thereof. [42] and [43] focus on SVRs in the context of time series data and find that they tend to outperform classical methods. [14] and [15] apply SVRs to predict the hourly demand for water in cities, an application similar to the UDP case.

In the SVR case, a linear function $\hat{y}_i = f(\vec{x}_i) = \langle \vec{w}, \vec{x}_i \rangle + b$ is fitted so that the actual labels y_i have a deviation of at most ϵ from their predictions $\hat{y}_i$ (cf., the constraints below). SVRs are commonly formulated as quadratic optimization problems as follows:

$$\text{minimize } \frac{1}{2} \|\vec{w}\|^2 + C \sum_{i=1}^{m} (\xi_i + \xi_i^*) \quad \text{subject to } \begin{cases} y_i - \langle \vec{w}, \vec{x}_i \rangle - b \leq \epsilon + \xi_i, \\ \langle \vec{w}, \vec{x}_i \rangle + b - y_i \leq \epsilon + \xi_i^* \end{cases}$$

$\vec{w}$ are the fitted weights in the row space of $\boldsymbol{X}$, b is a bias term in the column space of $\boldsymbol{X}$, and $\langle \cdot, \cdot \rangle$ denotes the dot product. By minimizing the norm of $\vec{w}$, the fitted function is flat and not prone to overfitting strongly. To allow individual samples outside the otherwise hard ϵ bounds, non-negative slack variables ξ_i and ξ_i^* are included. A non-negative parameter C regulates how many samples may violate the ϵ bounds and by how much. To model non-linear relationships, one could use a mapping $\Phi(\cdot)$ for the $\vec{x}_i$ from the row space of $\boldsymbol{X}$ to some higher dimensional space; however, as the optimization problem only depends on the dot product $\langle \cdot, \cdot \rangle$ and not the actual entries of $\vec{x}_i$, it suffices to use a kernel function k such that $k(\vec{x}_i, \vec{x}_j) = \langle \Phi(\vec{x}_i), \Phi(\vec{x}_j) \rangle$. Such kernels must fulfill certain mathematical properties, and, besides polynomial kernels, radial basis functions with $k(\vec{x}_i, \vec{x}_j) = exp(\gamma \|\vec{x}_i - \vec{x}_j\|^2)$ are a popular candidate where γ is a parameter controlling for how the distances between any two samples influence the final model. SVRs work well with sparse data in high dimensional spaces, such as intermittent demand data, as they minimize the risk of misclassification or predicting a significantly far off value by maximizing the error margin, as also noted by [3].

3. Model Formulation

In this section, we describe how the platform's raw data are pre-processed into model inputs and how the forecasting models are built and benchmarked against each other.

3.1. Overall Approach

On a conceptual level, there are three distinct aspects of the model development process. First, a pre-processing step transforms the platform's tabular order data into either time series in Sub-section 3.2 or feature matrices in Sub-section 3.6.4. Second, a benchmark methodology is developed in Sub-section 3.3 that compares all models on the same scale, in particular, classical models with ML ones. Concretely, the CV approach is adapted to the peculiar requirements of sub-daily and ordinal time series data. This is done to maximize the predictive power of all models into the future and to compare them on the same scale. Third, the forecasting models are described with respect to their assumptions and training requirements. Four classification dimensions are introduced:

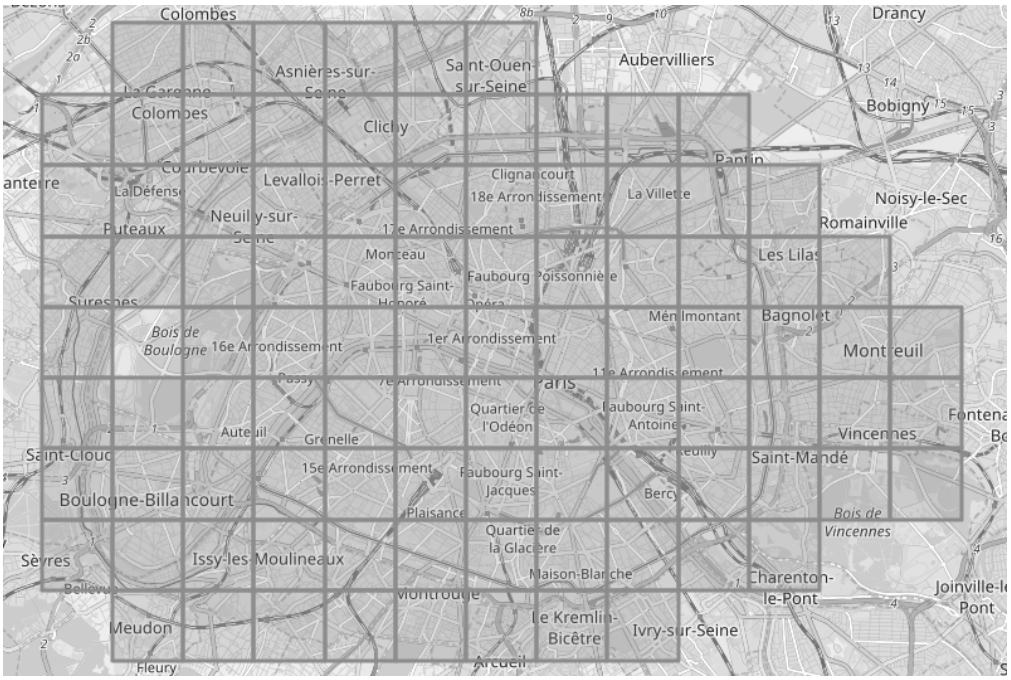
1. **Timeliness of the Information**: whole-day-ahead vs. real-time forecasts
2. **Time Series Decomposition**: raw vs. decomposed
3. **Algorithm Type**: "classical" statistics vs. ML
4. **Data Sources**: pure vs. enhanced (i.e., with external data)

Not all of the possible eight combinations are implemented; instead, the models are varied along these dimensions to show different effects and answer the research questions.

3.2. Gridification, Time Tables, and Time Series Generation

The platform's tabular order data are sliced with respect to both location and time and then aggregated into time series where an observation tells the number of orders in an area for a time step/interval. Figure 1 shows how the orders' delivery locations are each matched to a square-shaped cell, referred to as a pixel, on a grid covering the entire service area within a city. This gridification step is also applied to the pickup locations separately. The lower-left corner is chosen at random. [57] apply the same gridification idea and slice an urban area to model a location-routing problem, and [48] portray it as a standard method in the field of urban analytics. With increasing pixel sizes, the time series exhibit more order aggregation with a possibly stronger demand pattern. On the other hand, the larger the pixels, the less valuable become the generated forecasts as, for example, a courier sent to a pixel preemptively then faces a longer average distance to a restaurant in the pixel.

Figure 1: Gridification for delivery locations in Paris with a pixel size of 1 km^2



After gridification, the ad-hoc orders within a pixel are aggregated by their placement timestamps into sub-daily time steps of pre-defined lengths to obtain a time table as exemplified in Figure 2 with one-hour intervals.

Figure 2: Aggregation into a time table with hourly time steps

Time \ Day	...	Mon	Tue	Wed	Thu	Fri	Sat	Sun	...
11:00	...	$y_{11,Mon}$	$y_{11,Tue}$	$y_{11,Wed}$	$y_{11,Thu}$	$y_{11,Fri}$	$y_{11,Sat}$	$y_{11,Sun}$	...
12:00	...	$y_{12,Mon}$	$y_{12,Tue}$	$y_{12,Wed}$	$y_{12,Thu}$	$y_{12,Fri}$	$y_{12,Sat}$	$y_{12,Sun}$	...
...	...	...	...	...	...	...	...	...	...
20:00	...	$y_{20,Mon}$	$y_{20,Tue}$	$y_{20,Wed}$	$y_{20,Thu}$	$y_{20,Fri}$	$y_{20,Sat}$	$y_{20,Sun}$	...
21:00	...	$y_{21,Mon}$	$y_{21,Tue}$	$y_{21,Wed}$	$y_{21,Thu}$	$y_{21,Fri}$	$y_{21,Sat}$	$y_{21,Sun}$	...
...	...	...	...	...	...	...	...	...	...

Consequently, each $y_{t,d}$ in Figure 2 is the number of all orders within the pixel for the time of day t and day of week d (y_t and $y_{t,d}$ are the same but differ in that the latter acknowledges

a 2D view). The same trade-off as with gridification applies: The shorter the interval, the weaker is the demand pattern to be expected in the time series due to less aggregation while longer intervals lead to less usable forecasts. We refer to time steps by their start time, and their number per day, H, is constant. Given a time table as in Figure 2 there are two ways to generate a time series by slicing:

1. **Horizontal View**: Take only the order counts for a given time of the day
2. **Vertical View**: Take all order counts and remove the double-seasonal pattern induced by the weekday and time of the day with decomposition

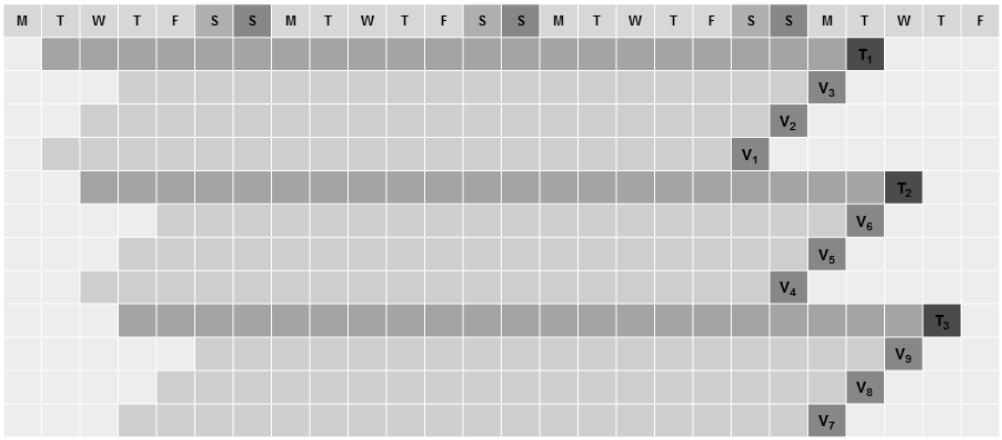
Distinct time series are retrieved by iterating through the time tables either horizontally or vertically in increments of a single time step. Another property of a generated time series is its length, which, following the next sub-section, can be interpreted as the sum of the production training set and the test day. In summary, a distinct time series is generated from the tabular order data based on a configuration of parameters for the dimensions pixel size, number of daily time steps H, shape (horizontal vs. vertical), length, and the time step to be predicted.

3.3. Unified Cross-Validation and Training, Validation, and Test Sets

The standard k-fold CV, which assumes no structure in the individual features of the samples, as shown in $\boldsymbol{X}$ above, is adapted to the ordinal character of time series data: A model must be evaluated on observations that occurred strictly after the ones used for training as, otherwise, the model knows about the future. Furthermore, some models predict only a single to a few time steps before being retrained, while others predict an entire day without retraining (cf., Sub-section 3.6.4). Consequently, we must use a unified time interval wherein all forecasts are made first before the entire interval is evaluated. As whole days are the longest prediction interval for models without retraining, we choose that as the unified time interval. In summary, our CV methodology yields a distinct best model per pixel and day to be forecast. Whole days are also practical for managers who commonly monitor, for example, the routing and thus the forecasting performance on a day-to-day basis. Our methodology assumes that the models are trained at least once per day. As we create operational forecasts

into the near future in this paper, retraining all models with the latest available data is a logical step.

Figure 3: Training, validation, and test sets during cross validation



The training, validation, and test sets are defined as follows. To exemplify the logic, we refer to Figure 3 that shows the calendar setup (i.e., weekdays on the x-axis) for three days T_1, T_2, and T_3 (shown in dark gray) for which we generate forecasts. Each of these days is, by definition, a test day, and the test set comprises all time series, horizontal or vertical, whose last observation lies on that day. With an assumed training horizon of three weeks, the 21 days before each of the test days constitute the corresponding training sets (shown in lighter gray on the same rows as T_1, T_2, and T_3). There are two kinds of validation sets, depending on the decision to be made. First, if a forecasting method needs parameter tuning, the original training set is divided into as many equally long series as validation days are needed to find stable parameters. The example shows three validation days per test day named V_n (shown in darker gray below each test day). The $21 - 3 = 18$ preceding days constitute the training set corresponding to a validation day. To obtain the overall validation error, the three errors are averaged. We call these *inner* validation sets because they must be repeated each day to re-tune the parameters and because the involved time series are true subsets of the original series. Second, to find the best method per day and pixel, the same averaging logic is applied on the outer level. For example, if we used two validation days to find the best method for T_3, we would average the errors of T_1 and T_2 for each method and select the winner; then, T_1 and T_2 constitute an *outer* validation set. Whereas the number of inner validation days

is method-specific and must be chosen before generating any test day forecasts in the first place, the number of outer validation days may be varied after the fact and is determined empirically as we show in Section 4.

Our unified CV approach is also optimized for large-scale production settings, for example, at companies like Uber. As [5] note, there is a trade-off as to when each of the inner time series in the example begins. While the forecasting accuracy likely increases with more training days, supporting inner series with increasing lengths, cutting the series to the same length allows caching the forecasts and errors. In the example, V_3, V_5, and V_7, as well as V_6 and V_8 are identical despite belonging to different inner validation sets. Caching is also possible on the outer level when searching for an optimal number of validation days for model selection. We achieved up to 80% cache hit ratios in our implementation in the empirical study, thereby saving computational resources by the same amount. Lastly, we assert that our suggested CV, because of its being unified around whole test days and usage of fix-sized time series, is also suitable for creating consistent learning curves and, thus, answering **Q3** on the relationship between forecast accuracy and amount of historic data: We simply increase the length of the outer training set holding the test day fixed. Thus, independent of a method's need for parameter tuning, all methods have the same demand history available for each test day forecast.

3.4. Accuracy Measures

Choosing an error measure for both model selection and evaluation is not straightforward when working with intermittent demand, as shown, for example, by [51], and one should understand the trade-offs between measures. [33] provide a study of measures with real-life data taken from the popular M3-competition and find that most standard measures degenerate under many scenarios. They also provide a classification scheme for which we summarize the main points as they apply to the UDP case:

1. **Scale-dependent Errors**: The error is reported in the same unit as the raw data. Two popular examples are the root mean square error (RMSE) and mean absolute error (MAE). They may be used for model selection and evaluation within a pixel, and are intuitively interpretable; however, they may not be used to compare errors of,

for example, a low-demand pixel (e.g., at the UDP's service boundary) with that of a high-demand pixel (e.g., downtown).

2. **Percentage Errors**: The error is derived from the percentage errors of individual forecasts per time step, and is also intuitively interpretable. A popular example is the mean absolute percentage error (MAPE) that is the primary measure in most forecasting competitions. Whereas such errors could be applied both within and across pixels, they cannot be calculated reliably for intermittent demand. If only one time step exhibits no demand, the result is a divide-by-zero error. This often occurs even in high-demand pixels due to the slicing.
3. **Relative Errors**: A workaround is to calculate a scale-dependent error for the test day and divide it by the same measure calculated with forecasts of a simple benchmark method (e.g., naïve method). An example could be $\text{RelMAE} = \text{MAE}/\text{MAE}_{\text{bm}}$. Nevertheless, even simple methods create (near-)perfect forecasts, and then MAE_{bm} becomes (close to) 0. These numerical instabilities occurred so often in our studies that we argue against using such measures.
4. **Scaled Errors**: [33] contribute this category and introduce the mean absolute scaled error (MASE). It is defined as the MAE from the actual forecasting method on the test day (i.e., "out-of-sample") divided by the MAE from the (seasonal) naïve method on the entire training set (i.e., "in-sample"). A MASE of 1 indicates that a forecasting method has the same accuracy on the test day as the (seasonal) naïve method applied on a longer horizon, and lower values imply higher accuracy. Within a pixel, its results are identical to the ones obtained with MAE. Also, we acknowledge recent publications, for example, [46] or [36], showing other ways of tackling the difficulties mentioned. However, only the MASE provided numerically stable results for all forecasts in our study.

Consequently, we use the MASE with a seasonal naïve benchmark as the primary measure in this paper. With the previously introduced notation, it is defined as follows:

$$\text{MASE} := \frac{\text{MAE}_{\text{out-of-sample}}}{\text{MAE}_{\text{in-sample}}} = \frac{\text{MAE}_{\text{forecasts}}}{\text{MAE}_{\text{training}}} = \frac{\frac{1}{H}\sum_{h=1}^{H}|y_{T+h} - \hat{y}_{T+h}|}{\frac{1}{T-k}\sum_{t=k+1}^{T}|y_t - y_{t-k}|}$$

The denominator can only become 0 if the seasonal naïve benchmark makes a perfect forecast

on each day in the training set except the first seven days, which never happened in our case study involving hundreds of thousands of individual model trainings. Further, as per the discussion in the subsequent Section 3.5, we also calculate peak-MASEs where we leave out the time steps of non-peak times from the calculations. For this analysis, we define all time steps that occur at lunch (i.e., noon to 2 pm) and dinner time (i.e., 6 pm to 8 pm) as peak. As time steps in non-peak times typically average no or very low order counts, a UDP may choose to not actively forecast these at all and be rather interested in the accuracies of forecasting methods during peaks only.

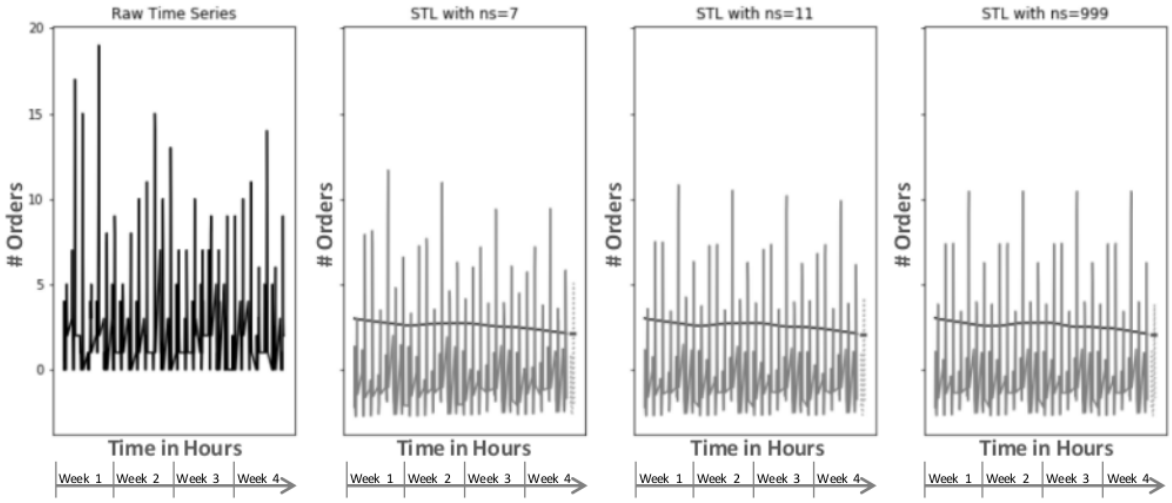
We conjecture that percentage error measures may be usable for UDPs facing a higher overall demand with no intra-day down-times in between but have to leave that to a future study. Yet, even with high and steady demand, divide-by-zero errors are likely to occur.

3.5. Time Series Decomposition

Concerning the time table in Figure 2, a seasonal demand pattern is inherent to both horizontal and vertical time series. First, the weekday influences if people eat out or order in with our partner receiving more orders on Thursday through Saturday than the other four days. This pattern is part of both types of time series. Second, on any given day, demand peaks occur around lunch and dinner times. This only regards vertical series. Statistical analyses show that horizontally sliced time series indeed exhibit a periodicity of $k = 7$, and vertically sliced series only yield a seasonal component with a regular pattern if the periodicity is set to the product of the number of weekdays and the daily time steps indicating a distinct intra-day pattern per weekday.

Figure 4 shows three exemplary STL decompositions for a 1 km^2 pixel and a vertical time series with 60-minute time steps (on the x-axis) covering four weeks: With the noisy raw data y_t on the left, the seasonal and trend components, s_t and t_t, are depicted in light and dark gray for increasing ns parameters. The plots include (seasonal) naïve forecasts for the subsequent test day as dotted lines. The remainder components r_t are not shown for conciseness. The periodicity is set to $k = 7 * 12 = 84$ as our industry partner has 12 opening hours per day.

Figure 4: STL decompositions for a medium-demand pixel with hourly time steps and periodicity $k = 84$



As described in Sub-section 2.1.3, with k being implied by the application, at the very least, the length of the seasonal smoothing window, represented by the ns parameter, must be calibrated by the forecaster: It controls how many past observations go into each smoothened s_t. Many practitioners, however, skip this step and set ns to a big number, for example, 999, then referred to as "periodic." For the other parameters, it is common to use the default values as specified in [16]. The goal is to find a decomposition with a regular pattern in s_t. In Figure 4, this is not true for $ns = 7$ where, for example, the four largest bars corresponding to the same time of day a week apart cannot be connected by an approximately straight line. On the contrary, a regular pattern in the most extreme way exists for $ns = 999$, where the same four largest bars are of the same height. This observation holds for each time step of the day. For $ns = 11$, s_t exhibits a regular pattern whose bars adapt over time: The pattern is regular as bars corresponding to the same time of day can be connected by approximately straight lines, and it is adaptive as these lines are not horizontal. The trade-off between small and large values for ns can thus be interpreted as allowing the average demand during peak times to change over time: If demand is intermittent at non-peak times, it is reasonable to expect the bars to change over time as only the relative differences between peak and non-peak times impact the bars' heights with the seasonal component being centered around 0. To confirm the goodness of a decomposition statistically, one way is to verify that r_t can

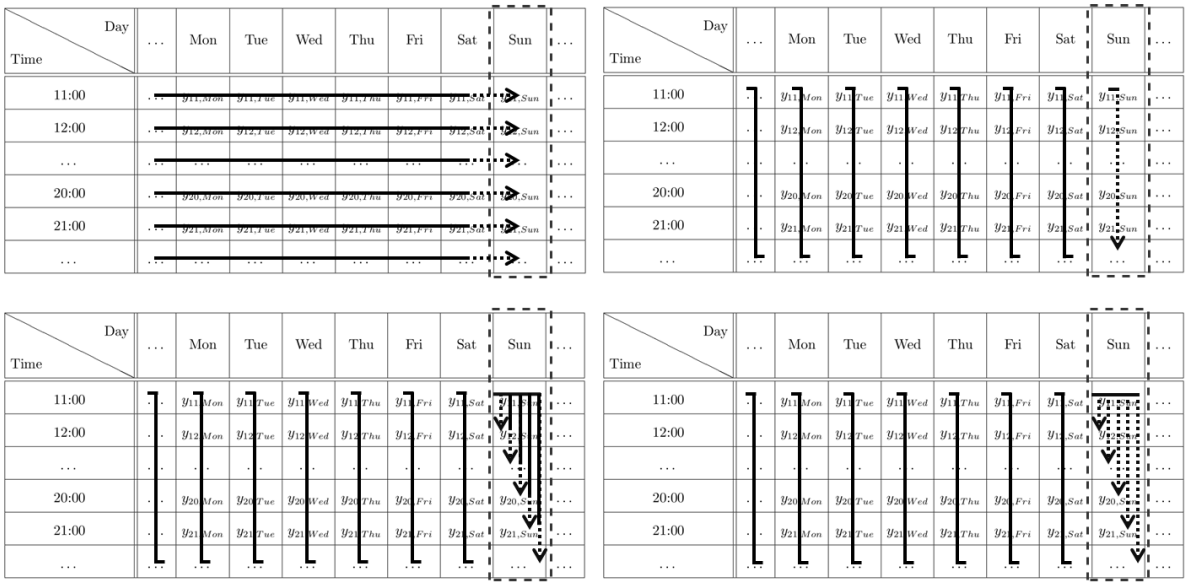
be modeled as a typical error process like white noise ϵ_t.

However, we suggest an alternative way of calibrating the STL method in an automated fashion based on our unified CV approach. As hinted at in Figure 4, we interpret an STL decomposition as a forecasting method on its own by just adding the (seasonal) naïve forecasts for s_t and t_t and predicting 0 for r_t. Then, the ns parameter is tuned just like a parameter for an ML model. To the best of our knowledge, this has not yet been proposed before. Conceptually, forecasting with the STL method can be viewed as a naïve method with built-in smoothing, and it outperformed all other benchmark methods in all cases.

3.6. Forecasting Models

This sub-section describes the concrete models in our study. Figure 5 shows how we classify them into four families with regard to the type of the time series, horizontal or vertical, and the moment at which a model is trained: Solid lines indicate that the corresponding time steps lie before the training, and dotted lines show the time horizon predicted by a model. For conciseness, we only show the forecasts for one test day. The setup is the same for each inner validation day.

Figure 5: Classification of the models by input type and training moment



3.6.1. Horizontal and Whole-day-ahead Forecasts.

The upper-left in Figure 5 illustrates the simplest way to generate forecasts for a test day before it has started: For each time of the day, the corresponding horizontal slice becomes the input for a model. With whole days being the unified time interval, each model is trained H times, providing a one-step-ahead forecast. While it is possible to have models of a different type be selected per time step, that did not improve the accuracy in the empirical study. As the models in this family do not include the test day's demand data in their training sets, we see them as benchmarks to answer **Q4**, checking if a UDP can take advantage of real-time information. The models in this family are as follows; we use prefixes, such as "h" here, when methods are applied in other families as well:

1. *naive*: Observation from the same time step one week prior
2. *trivial*: Predict 0 for all time steps
3. *hcroston*: Intermittent demand method introduced by [17]
4. *hholt*, *hhwinters*, *hses*, *hsma*, and *htheta*: Exponential smoothing without calibration
5. *hets*: ETS calibrated as described by [34]
6. *harima*: ARIMA calibrated as described by [32]

naive and *trivial* provide an absolute benchmark for the actual forecasting methods. *hcroston* is often mentioned in the context of intermittent demand; however, the method did not perform well at all. Besides *hhwinters* that always fits a seasonal component, the calibration heuristics behind *hets* and *harima* may do so as well. With $k = 7$, an STL decomposition is unnecessary here.

3.6.2. Vertical and Whole-day-ahead Forecasts without Retraining.

The upper-right in Figure 5 shows an alternative way to generate forecasts for a test day before it has started: First, a seasonally-adjusted time series a_t is obtained from a vertical time series by STL decomposition. Then, the actual forecasting model, trained on a_t, makes an H-step-ahead prediction. Lastly, we add the H seasonal naïve forecasts for the seasonal component s_t to them to obtain the actual predictions for the test day. Thus, only one training is required per model type, and no real-time data is used. By decomposing the raw time series, all long-term patterns are assumed to be in the seasonal component s_t, and

a_t only contains the level with a potential trend and auto-correlations. The models in this family are:

1. *fnaive*, *pnaive*: Sum of STL's trend and seasonal components' naïve forecasts
2. *vholt*, *vses*, and *vtheta*: Exponential smoothing without calibration and seasonal fit
3. *vets*: ETS calibrated as described by [34]
4. *varima*: ARIMA calibrated as described by [32]

As mentioned in Sub-section 3.3, we include the sum of the (seasonal) naïve forecasts of the STL's trend and seasonal components as forecasts on their own: For *fnaive*, we tune the "flexible" ns parameter, and for *pnaive*, we set it to a "periodic" value. Thus, we implicitly assume that there is no signal in the remainder r_t, and predict 0 for it. *fnaive* and *pnaive* are two more simple benchmarks.

3.6.3. Vertical and Real-time Forecasts with Retraining.

The lower-left in Figure 5 shows how models trained on vertical time series are extended with real-time order data as it becomes available during a test day: Instead of obtaining an H-step-ahead forecast, we retrain a model after every time step and only predict one step. The remainder is as in the previous sub-section, and the models are:

1. *rtholt*, *rtses*, and *rttheta*: Exponential smoothing without calibration and seasonal fit
2. *rtets*: ETS calibrated as described by [34]
3. *rtarima*: ARIMA calibrated as described by [32]

Retraining *fnaive* and *pnaive* did not increase accuracy, and thus we left them out. A downside of this family is the significant increase in computing costs.

3.6.4. Vertical and Real-time Forecasts without Retraining.

The lower-right in Figure 5 shows how ML models take real-time order data into account without retraining. Based on the seasonally-adjusted time series a_t, we employ the feature matrix and label vector representations from Sub-section 2.2.1 and set n to the number of daily time steps, H, to cover all potential auto-correlations. The ML models are trained once before a test day starts. For training, the matrix and vector are populated such that y_T is set to the last time step of the day before the forecasts, a_T. As the splitting during CV is

done with whole days, the ML models are trained with training sets consisting of samples from all times of a day in an equal manner. Thus, the ML models learn to predict each time of the day. For prediction on a test day, the H observations preceding the time step to be forecast are used as the input vector after seasonal adjustment. As a result, real-time data are included. The models in this family are:

1. *vrfr*: RF trained on the matrix as described
2. *vsvr*: SVR trained on the matrix as described

We tried other ML models such as gradient boosting machines but found only RFs and SVRs to perform well in our study. In the case of gradient boosting machines, this is to be expected as they are known not to perform well in the presence of high noise - as is natural with low count data - as shown, for example, by [39] or [41]. Also, deep learning methods are not applicable as the feature matrices only consist of several hundred to thousands of rows (cf., Sub-section 4). In Appendix A, we provide an alternative feature matrix representation that exploits the two-dimensional structure of time tables without decomposing the time series. In Appendix B, we show how feature matrices are extended to include predictors other than historical order data. However, to answer **Q5** already here, none of the external data sources improves the results in our study. Due to the high number of time series in our study, to investigate why no external sources improve the forecasts, we must us some automated approach to analyzing individual time series. [4] provide a spectral density estimation approach, called the Shannon entropy, that measures the signal-to-noise ratio in a database with a number normalized between 0 and 1 where lower values indicate a higher signal-to-noise ratio. We then looked at averages of the estimates on a daily level per pixel and find that including any of the external data sources from Appendix B always leads to significantly lower signal-to-noise ratios. Thus, we conclude that at least for the demand faced by our industry partner the historical data contains all of the signal.

4. Empirical Study: A Meal Delivery Platform in Europe

5. Conclusion

Glossary

CART Classification and Regression Trees. 10

CV Cross Validation. 9

fnaive "Flexible" STL Decomposition, with tuned ns parameter. 23

harima Autoregressive Integrated Moving Average Method, trained on horizontal time series. 22

hcroston Croston's Method, trained on horizontal time series. 22

hets ETS State Space Method, trained on horizontal time series. 22

hholt Holt's Linear Trend Method, trained on horizontal time series. 22

hhwinters Holt-Winter's Seasonal Method, trained on horizontal time series. 22

hses Simple Exponential Smoothing Method, trained on horizontal time series. 22

hsma Simple Moving Average Method, trained on horizontal time series. 22

htheta Theta Method, trained on horizontal time series. 22

MASE Mean Absolute Scaled Error. 18

ML Machine Learning. 3, 24

naive (Seasonal) Naïve Method. 22

pnaive "Periodic" STL Decomposition, with ns parameter set to large number. 23

RF Random Forest. 8

rtarima Autoregressive Integrated Moving Average Method, (re)trained on vertical time series. 23

rtets ETS State Space Method, (re)trained on vertical time series. 23

rtholt Holt's Linear Trend Method, (re)trained on vertical time series. 23

rtses Simple Exponential Smoothing Method, (re)trained on vertical time series. 23

rttheta Theta Method, (re)trained on vertical time series. 23

STL Seasonal and Trend Decomposition using Loess. 8

SVM Support Vector Machine. 11

SVR Support Vector Regression. 8

trivial Trivial Method. 22

UDP Urban Delivery Platform. 3

varima Autoregressive Integrated Moving Average Method, trained on vertical time series. 23

vets ETS State Space Method, trained on vertical time series. 23

vholt Holt's Linear Trend Method, trained on vertical time series. 23

vrfr Random Forest Regression Method, trained on vertical time series. 24

VRP Vehicle Routing Problem. 2

vses Simple Exponential Smoothing Method, trained on vertical time series. 23

vsvr Support Vector Regression Method, trained on vertical time series. 24

vtheta Theta Method, trained on vertical time series. 23

Appendix A. Tabular and Real-time Forecasts without Retraining

Regarding the structure of the feature matrix for the ML models in Sub-section 3.6.4, we provide an alternative approach that works without the STL method. Instead of decomposing a time series and arranging the resulting seasonally-adjusted time series a_t into a matrix $\boldsymbol{X}$, one can create a matrix with two types of feature columns mapped to the raw observations in $\vec{y}$: While the first group of columns takes all observations of the same time of day over a horizon of, for example, one week ($n_h = 7$), the second group takes all observations covering a pre-defined time horizon, for example 3 hours ($n_r = 3$ for 60-minute time steps), preceding the time step to be fitted. Thus, we exploit the two-dimensional structure of time tables as well, and conceptually model historical and recent demand. The alternative feature matrix appears as follows where the first three columns are the historical and the last three the recent demand features:

$$\vec{y} = \begin{pmatrix} y_T \\ y_{T-1} \\ \cdots \\ y_{1+n_h H} \end{pmatrix} \quad \boldsymbol{X} = \begin{bmatrix} y_{T-H} & y_{T-2H} & \cdots & y_{T-n_h H} & y_{T-1} & y_{T-2} & \cdots & y_{T-n_r} \\ y_{T-1-H} & y_{T-1-2H} & \cdots & y_{T-1-n_h H} & y_{T-2} & y_{T-3} & \cdots & y_{T-n_r-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{1+(n_h-1)H} & y_{1+(n_h-2)H} & \cdots & y_1 & y^*_{1+n_h H-1} & y^*_{1+n_h H-2} & \cdots & y^*_{1+n_h H-n_r} \end{bmatrix}$$

Being a detail, we note that the recent demand features lying on the end of the previous day are set to 0, which is shown with the * notation above. This alignment of the undecomposed order data y_t ensures that the ML models learn the two seasonal patterns independently. The parameters n_h and n_r must be adapted to the data, but we found the above values to work well.

As such matrices resemble time tables, we refer to them as tabular. However, we found the ML models with vertical time series to outperform the tabular ML models, which is why we disregarded them in the study. This tabular form could be beneficial for UDPs with a demand that exhibits a weaker seasonality such as a meal delivery platform.

Appendix B. Enhancing Forecasting Models with External Data

In this appendix, we show how the feature matrix in Sub-section 3.6.4 can be extended with features other than historical order data. Then, we provide an overview of what external data we tried out as predictors in our empirical study.

Appendix B.1. Enhanced Feature Matrices

Feature matrices can naturally be extended by appending new feature columns $x_{t,f}$ or x_f on the right where the former represent predictors changing throughout a day and the latter being static either within a pixel or across a city. f refers to an external predictor variable, such as one of the examples listed below. In the SVR case, the columns should be standardized before fitting as external predictors are most likely on a different scale than the historic order data. Thus, for a matrix with seasonally-adjusted order data a_t in it, an enhanced matrix looks as follows:

$$\vec{y} = \begin{pmatrix} a_T \\ a_{T-1} \\ \dots \\ a_{H+1} \end{pmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} a_{T-1} & a_{T-2} & \dots & a_{T-H} & x_{T,A} & \dots & x_B & \dots \\ a_{T-2} & a_{T-3} & \dots & a_{T-(H+1)} & x_{T-1,A} & \dots & x_B & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_H & a_{H-1} & \dots & a_1 & x_{H+1,A} & \dots & x_B & \dots \end{bmatrix}$$

Similarly, we can also enhance the tabular matrices from Appendix A. The same comments as for their pure equivalents in Sub-section 3.6.4 apply, in particular, that ML models trained with an enhanced matrix can process real-time data without being retrained.

Appendix B.2. External Data in the Empirical Study

In the empirical study, we tested four groups of external features that we briefly describe here.

Calendar Features:

- Time of day (as synthesized integers: e.g., 1,050 for 10:30 am, or 1,600 for 4 pm)
- Day of week (as one-hot encoded booleans)

- Work day or not (as booleans)

Features derived from the historical Order Data:

- Number of pre-orders for a time step (as integers)
- 7-day SMA of the percentages of discounted orders (as percentages): The platform is known for running marketing campaigns aimed at first-time customers at irregular intervals. Consequently, the order data show a wave-like pattern of coupons redeemed when looking at the relative share of discounted orders per day.

Neighborhood Features:

- Ambient population (as integers) as obtained from the ORNL LandScan database
- Number of active platform restaurants (as integers)
- Number of overall restaurants, food outlets, retailers, and other businesses (as integers) as obtained from the Google Maps and Yelp web services

Real-time Weather (raw data obtained from IBM's Wunderground database):

- Absolute temperature, wind speed, and humidity (as decimals and percentages)
- Relative temperature with respect to 3-day and 7-day historical means (as decimals)
- Day vs. night defined by sunset (as booleans)
- Summarized description (as indicators -1, 0, and $+1$)
- Lags of the absolute temperature and the summaries covering the previous three hours

Unfortunately, we must report that none of the mentioned external data improved the accuracy of the forecasts. Some led to models overfitting the data, which could not be regulated. Manual tests revealed that real-time weather data are the most promising external source. Nevertheless, the data provided by IBM's Wunderground database originate from weather stations close to airports, which implies that we only have the same aggregate weather data for the entire city. If weather data is available on a more granular basis in the future, we see some potential for exploitation.

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